

Quintessence with $O(N)$ Symmetry

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ABSTRACT: We study a new class of quintessence models, in which the scalar field possesses a $O(N)$ internal symmetry. We give a critical condition of instability for the potential against Q ball formation. We find that the most widely used potentials of quintessence don't satisfy the above condition, and therefore the $O(N)$ quintessence with these potential will not lead to the Q ball formation. It is worth noting that $O(N)$ quintessence with cosine-type potential is especially interesting in that the angular contribution is not negligible.

1. Introduction

Recent observations on the spectrum of CMB anisotropies[1] indicate that about 70 percent of the total energy in the universe should be hidden as dark energy[2], although these observations should be extended to even larger red-shifts and smaller angles, as several missions under preparation will do in the near future. Observations of type Ia supernova(SNIa)[3] shows that the expansion of universe is accelerating and therefore requires that the equation-of-state parameter $w = \frac{p}{\rho}$ of the total fluid should be necessarily smaller than $-\frac{1}{3}$, where p and ρ are the pressure and energy density of the fluid in universe respectively. Hitherto, two possible candidates for dark energy have been suggested. One is the existence of a cosmological constant and another is a time-varying scalar field "quintessence"[4, 5], which has caught much attention ever since its invention. The quintessence models can be classified into tracker-type[6] and cosine-type[7, 8]. The tracker-type quintessence has an attractor-like solution which explains the current dark energy without fine-tuning the initial condition, while the cosine type one needs tuning of the initial value of the scalar field although it has been shown that the fine-tuning problem could be alleviated in the extended models[9]. This is why most astrophysicist prefer the tracker quintessence to the cosine one. In fact, even in tracker-type model, the amplitude of the field and the parameters in the potential should be carefully adjusted in order to meet the experimental results. It is worth noting that many theorists tried to make the potential for tracker-type quintessence arise from particle physics naturally, even if this issue is really hard [10]. But the potentials for cosine-type models can arise rather naturally from particle physics, which attract some theorists' attention.

One of the most important observable effects of quintessence is the CMB anisotropy and the different types of quintessence models will eventually lead to different spectrum of the anisotropy. Therefore, more accurate measurements[11, 12] of the CMB anisotropy (including the forthcoming measurements made by MAP[13], PLANCK[14]) will lay more strict restrictions on quintessence models, confirming some of them and abandoning the rest.

A new generalization to quintessence has been proposed by Boyle et al[15] and Gu and Hwang[16]. In their models, The slowly evolving scalar field of the quintessence is replaced by a complex scalar field that is spinning in a circular orbit in a $U(1)$ -symmetric potential. The corresponding equation of state parameter $w = \frac{p}{\rho}$ for the spinning quintessence, or spintessence as they call it, has been discussed and a possible way to distinguish spintessence from other quintessence scenarios by means of future observations has been suggested too. It is shown that a complex scalar field can be regarded as ordinary quintessence if the field does not rotate at all, and will be spintessence when it rotates rapidly in the potential.

In this paper, we make a further generalization to spintessence by replacing the complex scalar field with a N -plet scalar field which is spinning in a $O(N)$ -symmetric potential. When one of the angular components is fixed, this $O(N)$ quintessence model will reduce to the $O(N-1)$ quintessence model. If the $N-2$ angular components are fixed, $O(N)$ quintessence model will reduce to the $U(1)$ spintessence model mentioned above. If all angular components are fixed, $O(N)$ quintessence model will reduce to the quintessence. Especially, the attractor property of the $O(N)$ quintessence is still held for some typical quintessence models and an example is shown in this paper. It is well known that the scalar field with negative pressure are generally unstable, the fluctuations may grow rapidly and go nonlinear to form Q balls[18, 19, 20]. Once the Q balls are produced, they will act as a dark matter, then the energy density evolves as a^{-3} [15]. In fact, almost all the charges of the field are absorbed into the produced Q balls so that there is no homogeneous field left to be dark energy[15]. This is against the purpose we propose the quintessence. In this paper,

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we give a critical condition of instability for the $O(N)$ quintessence. Under this condition, the Q ball formation scenario will occur. Fortunately, we found that the most widely used potentials for quintessence (Note: some tracker-type potentials are not stable against Q ball formation at $\lambda R = O(1)$ epoch) don't satisfy the above condition, and therefore the $O(N)$ quintessence with these potentials are generally stable.

It is worth noting that this generalization ($O(N)$ quintessence) is not very appealing for the tracker-type quintessence at the late time, for the amplitude of the field will increase to be very large with the time evolution and the contribution from the angular component ($\frac{\Omega^2}{a^6 R^3}$, where R is the amplitude of the field) to the equation of motion becomes negligible as time evolves; but in the early epoch it will be very important for the contribution from the "angular" component is not negligible. Furthermore, for the cosine-type potential, since the amplitude of the quintessence field does not necessarily increase to be very large to accelerate the expansion of the universe, and the angular contribution is important for the whole evolution. Therefore, it is interesting to discuss $O(N)$ quintessence in both tracker-type models and cosine-type models.

2. $O(N)$ quintessence

We start from the flat Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\alpha^2 + r^2 \sin^2 \alpha d\beta^2) \quad (1)$$

The Lagrangian density for the quintessence with $O(N)$ symmetry is

$$L_\Phi = \frac{1}{2} g^{\mu\nu} (\partial_\mu \Phi^a) (\partial_\nu \Phi^a) - V(|\Phi^a|) \quad (2)$$

where Φ^a is the component of the scalar field, $a = 1, 2, \dots, N$. To make it possess a $O(N)$ symmetry, we write it in the following form

$$\begin{aligned} \Phi^1 &= R(t) \cos \varphi_1(t) \\ \Phi^2 &= R(t) \sin \varphi_1(t) \cos \varphi_2(t) \\ \Phi^3 &= R(t) \sin \varphi_1(t) \sin \varphi_2(t) \cos \varphi_3(t) \\ &\dots\dots\dots \\ \Phi^{N-1} &= R(t) \sin \varphi_1(t) \dots \sin \varphi_{N-2}(t) \cos \varphi_{N-1}(t) \\ \Phi^N &= R(t) \sin \varphi_1(t) \dots \sin \varphi_{N-2}(t) \sin \varphi_{N-1}(t) \end{aligned} \quad (3)$$

Therefore, we have $|\Phi^a| = R$ and assume that the potential of the $O(N)$ quintessence depends only on R . The Lagrangian density then take the following form:

$$\begin{aligned} L_\Phi &= \frac{1}{2} (\dot{R}^2 + R^2 \dot{\varphi}_1^2 + R^2 \sin^2 \varphi_1 \dot{\varphi}_2^2 + \dots \\ &\quad + R^2 \sin^2 \varphi_1 \sin^2 \varphi_2 \dots \sin^2 \varphi_{N-2} \dot{\varphi}_{N-1}^2) - V(R) \end{aligned} \quad (4)$$

where the dot denotes the derivative with respect to t .

The action for the universe is :

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{16\pi G} R_s - \rho_M + L_\Phi \right) \quad (5)$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, G is the Gravitational constant, R_s is the Ricci scalar, and ρ_M is the non-relativistic matter density. Using the metric tensor (1) and action (5), we can obtain the equation of motion for $\varphi_1, \dots, \varphi_{N-1}$ as follows:

$$\ddot{\varphi}_1 + (2\frac{\dot{R}}{R} + 3H)\dot{\varphi}_1 - \cos \varphi_1 \sin \varphi_1 (\dot{\varphi}_2^2 + \sin^2 \varphi_2 \dot{\varphi}_3^2 + \dots + \sin^2 \varphi_2 \sin^2 \varphi_3 \dots \sin^2 \varphi_{N-2} \dot{\varphi}_{N-1}^2) = 0 \quad (6)$$

$$\ddot{\varphi}_2 + (2\frac{\dot{R}}{R} + 3H + 2 \cot \varphi_1 \dot{\varphi}_1)\dot{\varphi}_2 - \cos \varphi_2 \sin \varphi_2 (\dot{\varphi}_3^2 + \sin^2 \varphi_3 \dot{\varphi}_4^2 + \dots + \sin^2 \varphi_3 \sin^2 \varphi_4 \dots \sin^2 \varphi_{N-2} \dot{\varphi}_{N-1}^2) = 0 \quad (7)$$

$$\ddot{\varphi}_{N-2} + (2\frac{\dot{R}}{R} + 3H + 2 \cot \varphi_1 \dot{\varphi}_1 + \dots + 2 \cot \varphi_{N-3} \dot{\varphi}_{N-3})\dot{\varphi}_{N-2} - \cos \varphi_{N-2} \sin \varphi_{N-2} \dot{\varphi}_{N-1}^2 = 0 \quad (8)$$

$$\ddot{\varphi}_{N-1} + (2\frac{\dot{R}}{R} + 3H + 2 \cot \varphi_1 \dot{\varphi}_1 + \dots + 2 \cot \varphi_{N-2} \dot{\varphi}_{N-2})\dot{\varphi}_{N-1} = 0 \quad (9)$$

we have $N - 1$ independent first integral of the system of equations

$$\dot{\varphi}_1 = (\Omega^2 - \frac{\Omega_1^2}{\sin^2 \varphi_1})^{\frac{1}{2}} (a^3 R^2)^{-1} \quad (10)$$

$$\dot{\varphi}_2 = (\Omega_1^2 - \frac{\Omega_2^2}{\sin^2 \varphi_2})^{\frac{1}{2}} (a^3 R^2 \sin^2 \varphi_1)^{-1} \quad (11)$$

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$$\dot{\varphi}_{N-2} = (\Omega_{N-3}^2 - \frac{\Omega_{N-2}^2}{\sin^2 \varphi_{N-2}})^{\frac{1}{2}} (a^3 R^2 \sin^2 \varphi_1 \dots \sin^2 \varphi_{N-3})^{-1} \quad (12)$$

$$\dot{\varphi}_{N-1} = \Omega_{N-2} (a^3 R^2 \sin^2 \varphi_1 \dots \sin^2 \varphi_{N-2})^{-1} \quad (13)$$

where $\Omega, \Omega_1, \dots, \Omega_{N-2}$ are $N-1$ independent constants determined by the initial condition of $\varphi_i, i = 1, 2, \dots, N-1$. The Einstein equations and the radial equation of scalar fields can be written as

$$H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3} [\rho_M + \rho_\Phi] \quad (14)$$

$$(\frac{\ddot{a}}{a}) = -\frac{8\pi G}{3} [\frac{1}{2}\rho_M + 2p_\Phi + V(R)] \quad (15)$$

$$\ddot{R} + 3H\dot{R} - \frac{\Omega^2}{a^6 R^3} + \frac{\partial V(R)}{\partial R} = 0 \quad (16)$$

where

$$\rho_\Phi = \frac{1}{2}(\dot{R}^2 + \frac{\Omega^2}{a^6 R^2}) + V(R) \quad (17)$$

$$p_\Phi = \frac{1}{2}(\dot{R}^2 + \frac{\Omega^2}{a^6 R^2}) - V(R) \quad (18)$$

are the energy density and pressure of the Φ field respectively, and H is Hubble parameter. The equation-of-state parameter for the $O(N)$ quintessence is

$$w = \frac{p_\Phi}{\rho_\Phi} = \frac{\dot{R}^2 + \frac{\Omega^2}{a^6 R^2} - 2V(R)}{\dot{R}^2 + \frac{\Omega^2}{a^6 R^2} + 2V(R)} \quad (19)$$

For the $O(N)$ quintessence to accelerate the expansion of universe, its equation-of-state parameter must satisfy $w < -\frac{1}{3}$ which is equivalent to

$$\dot{R}^2 + \frac{\Omega^2}{a^6 R^2} < V(R) \quad (20)$$

where the term $\frac{\Omega^2}{a^6 R^2}$ comes from the "total angular motion". The most prominent feature of $O(N)$ quintessence is that it will not reduce to a cosmological constant even when the R is spatially uniform and time-independent.

It is worth noting that the introduction of the "angular" component in $O(N)$ quintessence will not change the attractor property of the dynamical system for the angular part will decrease rapidly with the increase of a and R . In order to make it more clear, we will show this property through a specific model in which, we choose the quintessence potential as the widely studied tracker potential $V(R) = V_0 \exp(-\lambda \kappa R)$. Following Ref.[17] the system of equations are:

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_M + p_M + \dot{R}^2 + \frac{\Omega^2}{a^6 R^2}) \quad (21)$$

$$\dot{\rho}_M = -3H(\rho_M + p_M) \quad (22)$$

$$\ddot{R} + 3H\dot{R} - \frac{\Omega^2}{a^6 R^3} - \lambda \kappa V(R) = 0 \quad (23)$$

$$H^2 = \frac{\kappa^2}{3}[\rho_M + \frac{1}{2}(\dot{R}^2 + \frac{\Omega^2}{a^6 R^2}) + V(R)] \quad (24)$$

Here ρ_M and p_M are the energy density and pressure of the non-relativistic matter and $p_M = (\gamma - 1)\rho_M$, γ is a constant, $0 \leq \gamma \leq 2$ and $\kappa^2 = 8\pi G$. Now, introducing the following variables: $x = \frac{\kappa}{\sqrt{6}H}$, $y = \frac{\kappa\sqrt{V(R)}}{\sqrt{3}H}$, $z = \frac{\kappa}{\sqrt{6}H} \frac{\Omega}{a^3 R}$, $\xi = \frac{1}{\kappa R}$ and $N = \log a$, the Eqs.21-24 become the following autonomous system:

$$\frac{dx}{dN} = \frac{3}{2}x[\gamma(1 - x^2 - y^2 - z^2) + 2(x^2 + z^2)] - (3x - \sqrt{6}z^2\xi - \sqrt{\frac{3}{2}}\lambda y^2) \quad (25)$$

$$\frac{dy}{dN} = \frac{3}{2}y[\gamma(1 - x^2 - y^2 - z^2) + 2(x^2 + z^2)] - \sqrt{\frac{3}{2}}\lambda xy \quad (26)$$

$$\frac{dz}{dN} = -3z - \frac{3}{2}z[\gamma(1 - x^2 - y^2 - z^2) + 2(x^2 + z^2)] - \sqrt{6}xz\xi \quad (27)$$

$$\frac{d\xi}{dN} = -\sqrt{6}\xi^2 x \quad (28)$$

It is not difficult to obtain the critical points of the above autonomous system as: $(x, y, z, \xi) = (0, 0, 0, 0)$, $(-1, 0, 0, 0)$, $(1, 0, 0, 0)$, $(\frac{\lambda}{\sqrt{6}}, \sqrt{1 - \frac{\lambda^2}{6}}, 0, 0)$ and $(\sqrt{\frac{3}{2}}\frac{\gamma}{\lambda}, \sqrt{\frac{3\gamma(1-\gamma)}{\lambda^2}}, 0, 0)$. These critical points will reduce to the case discussed in

the ordinary quintessence model[17] and the "angular" component will not alter the attractor property because the vanish of "angular" contribution ($z = 0$).

3. Quintessence without Q ball formation scenario

3.1 Condition for Q ball formation

As some authors have shown in the context of the Affleck-Dine baryogenesis [18, 19], the formation of Q balls is very common for the scalar fields with $O(N)$ symmetry. However, this is not favorable for $O(N)$ quintessence. In the following, we will give a condition of instability, under which the the $O(N)$ quintessence model will lead to the formation of Q balls. For simplicity, we consider only the epoch that quintessence become dominant.

Follow Ratra and Peebles[4], we will carry out our investigation in synchronous gauge and linearize the metric about a spatially flat FRW background. The line element is as follow,

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} - h_{ij})dx^i dx^j \quad (29)$$

where h_{ij} are the metric fluctuations and $|h_{ij}| \ll 1$. For simplicity, we write the first-order equations of perturbations for the $N = 3$ case

$$\begin{aligned} &(\delta\ddot{R}) - \frac{1}{a^2}\nabla^2(\delta R) - [\dot{\varphi}_1^2 + \sin^2\varphi_1](\delta R) + 3\frac{\dot{a}}{a}(\delta\dot{R}) \\ &+ V''(R)(\delta R) - \frac{1}{2}\dot{h}\dot{R} - 2\dot{\varphi}_1 R(\delta\dot{\varphi}_1) \\ &- R\sin(2\varphi_1)\dot{\varphi}_2^2(\delta\varphi_1) - 2R\sin^2(\varphi_1)\dot{\varphi}_2(\delta\dot{\varphi}_2) = 0 \end{aligned} \quad (30)$$

$$\begin{aligned} &(\delta\ddot{\varphi}_1) - \frac{1}{a^2}\nabla^2(\delta\varphi_1) + 3\frac{\dot{a}}{a}(\delta\dot{\varphi}_1) + 2\frac{\dot{R}}{R}(\delta\dot{\varphi}_1) \\ &+ \dot{\varphi}_2^2 \cos(2\varphi_1)(\delta\varphi_1) - \frac{1}{2}\dot{h}\dot{\varphi}_1 - \sin(2\varphi_1)\dot{\varphi}_2(\delta\dot{\varphi}_2) \\ &+ \frac{2}{R}\dot{\varphi}_1(\delta\dot{R}) - \frac{2}{R^2}\dot{R}\dot{\varphi}_1(\delta R) = 0 \end{aligned} \quad (31)$$

$$\begin{aligned} &(\delta\ddot{\varphi}_2) - \frac{1}{a^2}\nabla^2(\delta\varphi_2) + 3\frac{\dot{a}}{a}(\delta\dot{\varphi}_2) + 2\frac{\dot{R}}{R}(\delta\dot{\varphi}_2) - \frac{1}{2}\dot{h}\dot{\varphi}_2 \\ &+ 2\cot(\varphi_1)[\dot{\varphi}_1(\delta\varphi_2) + \dot{\varphi}_2(\delta\dot{\varphi}_1)] - 2\csc^2(\varphi_1)\dot{\varphi}_1\dot{\varphi}_2(\delta\varphi_1) \\ &+ \frac{2}{R}\dot{\varphi}_2(\delta\dot{R}) - \frac{2}{R^2}\dot{R}\dot{\varphi}_2(\delta R) = 0 \end{aligned} \quad (32)$$

It is necessary to point out that we here discuss mainly the stability against Q ball formation, which require that the fluctuations of the scalar field should not violate the internal $O(N)$ symmetry. That is, only the "radial" part of the scalar field is perturbed. If the "angular" parts of the scalar field are perturbed too, then the internal symmetry will not hold any longer and thus the Q balls will not form even if the fluctuation is not damped. Therefore, we consider the perturbation that can preserve the $O(N)$ symmetry of the scalar field as following:

$$\begin{aligned} R(t, \mathbf{x}) &= R_0(t) + \delta R(t, \mathbf{x}) \\ \varphi_1(t, \mathbf{x}) &= \varphi_1(t) \\ &\dots\dots\dots \\ \varphi_{N-1}(t, \mathbf{x}) &= \varphi_{N-1}(t) \end{aligned} \quad (33)$$

Then the first order equations of motion for the radial fluctuation of the scalar field and metric fluctuations are

$$\begin{aligned}
(\ddot{\delta R}) + 3H(\dot{\delta R}) - \frac{1}{a^2}\nabla^2(\delta R) + \frac{\Omega^2}{a^6 R_0^4}(\delta R) \\
+ V''(R_0)(\delta R) - \frac{1}{2}\dot{h}\dot{R}_0 = 0
\end{aligned} \tag{34}$$

$$\ddot{h} + 2H\dot{h} = 2\dot{R}_0(\dot{\delta R}) + 2\frac{\Omega^2}{a^6 R_0^4}(\delta R) - V'(R_0)(\delta R) \tag{35}$$

$$\dot{h}_{,i} - \dot{h}_{ij,j} = \dot{R}_0 \partial_i(\delta R) \tag{36}$$

$$\begin{aligned}
\frac{1}{a^2}(h_{ij,kk} + h_{,ij} - h_{ik,jk} - h_{jk,ik}) - 3H\dot{h}_{ij} \\
- H\dot{h}\delta_{ij} - \ddot{h}_{ij} = \delta_{ij}V'(R_0)(\delta R)
\end{aligned} \tag{37}$$

where h is the trace of h_{ij} . If we choose $\Omega = 0$, i.e. in the case of $N = 1$ the Eqs.(34)-(37) will reduce to Ratra-Peebles's case in the absence of baryonic term[4]. Since the equations of motion for the metric and scalar fluctuations(Eq.(26) and Eq.(27)) are linear equations, the fluctuations could be taken as the following form

$$\delta R(t, \mathbf{x}) = \delta R_0 \exp(\alpha t + i\mathbf{k}\mathbf{x}) \tag{38}$$

$$h(t, \mathbf{x}) = h_0 \exp(\alpha t + i\mathbf{k}\mathbf{x}) \tag{39}$$

then for nontrivial δR_0 and h_0 , we have

$$\alpha^2 + (3H - \frac{1}{2}\frac{h_0}{\delta R_0}\dot{R}_0)\alpha - \frac{\Omega^2}{a^6 R_0^4} + \frac{k^2}{a^2} + \frac{\partial^2 V}{\partial R_0^2} = 0 \tag{40}$$

If α is real and positive, the fluctuations will grow exponentially, and go nonlinear to form Q balls. Therefore the instability band for this fluctuation is

$$0 < k^2 < \frac{\Omega^2}{a^4 R^4} - \frac{a^2 \partial^2 V}{\partial R^2} \tag{41}$$

From Eq.(20) and Eq.(41), it is easily obtained that the instability band will not exist if the potentials of the $O(N)$ quintessence satisfy

$$\frac{\partial^2 V(R)}{\partial R^2} > \frac{V(R)}{R^2} \tag{42}$$

Eq.(42) is a critical condition for the potential of $O(N)$ quintessence, under which the formation of Q balls will not occur and thus can be used to determine whether the potential is "appropriate" for the $O(N)$ quintessence model. In the following, we will show that the two types of quintessence models all satisfy the above condition.

3.2 The tracker-type quintessence

Tracker-type quintessence models, as previously mentioned, have an attractor-like solution in a sense that a very wide range of initial conditions converge to a common evolving track. One can call the epoch when this attractor-like solution is realized as "tracking regime", in which the ratio of kinetic energy to potential energy is almost constant. In fact, the tracker-type quintessence can also be categorized into two different cases: case A and case B. For case A,

the energy density of the quintessence is not a significant fraction of the total energy density of the universe at the early epoch. On the other hand, the energy density of the quintessence can be significant at early epoch for the case B.

It is not difficult to prove that the most widely investigated potentials for case A (such as the inverse power law potential $V(R) = V_1(\frac{R_0}{R})^n$, which was originally studied in Ref.[4] and later was investigated in Ref.[21], and the potential $V(R) = V_0[\exp(\frac{R_0}{R}) - 1]$ that has been studied in Ref.[22]) all satisfy the Eq.(42). In the following, we take the potential $V(R) = V_0\left[\exp(\frac{R_0}{R}) - 1\right]$ as an example and show that it satisfies Eq.(42). Let

$$f(\frac{R_0}{R}) = \frac{R^2}{V_0} \left[\frac{\partial^2 V(R)}{\partial R^2} - \frac{V(R)}{R^2} \right] \quad (43)$$

and denote $x = \frac{R_0}{R}$, then we have $f(x) = (x^2 + 2x - 1)e^x + 1$, when $R \rightarrow \infty$, $x \rightarrow 0$. Since $f(x)$ is a monotonous increasing function of x , and $f(0) = 0$, then for all finite R , we have $f(x) > 0$, that is, the potential $V(R) = V_0\left[\exp(\frac{R_0}{R}) - 1\right]$ satisfy the condition(42). Therefore, $O(N)$ quintessence models with the above potentials will not lead to Q ball formation.

As an important example of case B, the potential can be chosen as [22]

$$V(R) = g(R)e^{-\lambda R} \quad (44)$$

where $g(R)$ varies more slowly than $e^{-\lambda R}$ in the tracking regime. The stability condition(42) is reduced to

$$\lambda^2 - 2\lambda \frac{g'}{g} + \frac{g''}{g} > \frac{1}{R^2} \quad (45)$$

For the simplest case $g(R) = R^2$, we find that quintessence field is unstable at earlier epoch of universe for the amplitudes satisfying $2 - \sqrt{3} < \lambda R < 2 + \sqrt{3}$ in the tracking regime. This means that case B of tracker $O(N)$ quintessence is unstable during $\lambda R = O(1)$ epoch. But if $\lambda R < 2 - \sqrt{3}$ or $\lambda R > 2 + \sqrt{3}$, the model is stable against Q ball formation. In fact, the above result does not depend on the detailed structure of the function $g(R)$ as long as $g(R)$ is a slowly varying function in the tracking regime. Thus, the same argument is applicable to case B with $V(R) = g(R)e^{-\lambda R}$.

3.3 The cosine-type quintessence

The potential for Cosine-type quintessence is

$$V(R) = \Lambda^4 \left[1 - \cos\left(\frac{R}{R_0}\right) \right] \quad (46)$$

which can be generated if the quintessence is pseudo Nambu-Goldstone boson field. Firstly, we investigate the evolution of $O(N)$ quintessence in such a potential. There are three parameters Λ , R_0 , and R_I (R_I denotes the initial value of the field) in the cosine-type potential. Notice that there is no possible value of R_I consistent with the dark energy that will account 70 percent energy density for $\Lambda \lesssim 1.9 \times 10^{-12} \text{ GeV}$, because the ρ_Φ is at most $2\Lambda^4$. When $2\Lambda^4$ is close to $0.7\rho_c$ (ρ_c is the critical density of the present Universe), R_I should be close to πR_0 to realize the relevant value of $0.7\rho_c$. If the slow-roll condition is satisfied until very recently, the energy density of the quintessence field becomes dominant and its equation of state parameter w is close to -1. However, it is not difficult to obtain that the value of w is very large at the early epoch. Therefore, there should be a critical value of radius of the Universe a_c . When $a < a_c$, the expansion of the Universe is not accelerated,

$$\frac{a_c}{a_0} \approx \left[\frac{2(1 + w_0)}{1 - w_0} \right]^{\frac{1}{6}} \quad (47)$$

where, a_0 and w_0 are the present value of scale factor of the Universe and w respectively. We can read off this behavior from the fig1.

In contrast, for larger Λ , R_I smaller than πR_0 is possible and the motion of the scalar field becomes important. At the beginning, the term of total angular momentum is dominant and the kinetic energy term $\frac{1}{2}\dot{R}^2$ can be neglected. But, with the expansion of the Universe, the kinetic energy term and potential energy term become more and more important and dominant in the end. At the later stage, if we suppose further that $H < \frac{\Lambda^2}{R_0}$, then the parameter w will oscillate between -1 and +1. Typical behaviors of the parameter w as a function of the scale factor are also shown in the fig1.

In this class of models, effective mass of the quintessence field is always $O(\frac{\Lambda^2}{R_0})$ and is insensitive to the amplitude of R . If one assumes that (i)the current quintessence field satisfies the slow-roll condition, i.e. $w \simeq -1$; (ii) $\frac{\Lambda^2}{R_0}$ is not far greater than the H of the present universe, then the quintessence field is negligible when $z \gg 1$ (z is the redshift). Therefore the present energy density of the quintessence field is sensitively depends on the initial amplitude of R_I in the Cosine-type model. The condition Eq(42)can be rewritten as:

$$\left[\left(\frac{R}{R_0} \right)^2 + 1 \right] \cos\left(\frac{R}{R_0} \right) > 1 \quad (48)$$

One can find that the stable condition (42) is satisfied when $0 < R < \mu_c R_0$, where critical value $\mu_c \approx 1.1025$.

Under this circumstance, the variation of the quintessence field is almost negligible, which make it behave like the cosmological constant. However, R_I could be smaller than πR_0 when Λ is very large, and the kinetic energy of the quintessence becomes important. In particular, the quintessence will oscillate around the minimum of the potential and thus satisfy the satability condition(42) if Λ becomes large enough. That is to say, for the $O(N)$ quintessence to be stable and can accelerate the expansion of the universe, the amplitude of R is not necessarily goes to be very large and therefore the angular contribution can not be neglected. In the $N = 1$ case, there are no questions concerning the Q ball formation. If Λ is close to $1.9 \times 10^{-12}\text{GeV}$, then the $N = 1$ is the only choice due to the stability constraint. But if Λ is greater than $1.9 \times 10^{-12}\text{GeV}$, the $O(N)$ model will be possibly stable against Q ball formation. Since different Λ will lead to different CMB anisotropies which may be detectable in the future satellite experiments, one can decide whether the cosine-type quintessence is consist of one scalar field or many fields with an symmetry constrains by experiments.

4. Conclusion and discussion

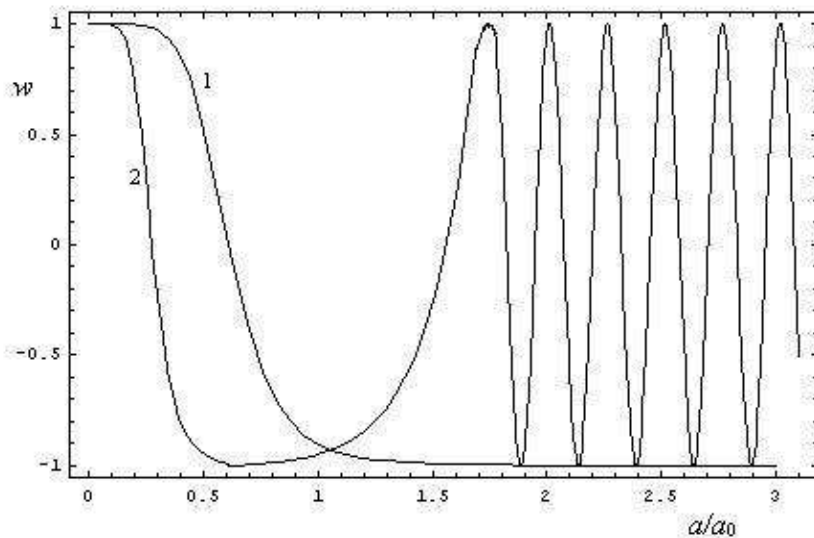


FIG. 1: Evolution of the parameter w with respect to $\frac{a}{a_0}$ for (i) $\Lambda = 1.9 \times 10^{-12}\text{GeV}$ (curve 1), and (ii) $4.0 \times 10^{-12}\text{GeV}$ (curve 2) respectively. Here, we take $R_0 = 5.0 \times 10^{17}\text{GeV}$, $w_0 = -0.9$ and the Hubble constant $H_0 = 65\text{km}\cdot\text{sec}^{-1}\cdot\text{Mpc}^{-1}$.

We have studied a class of new quintessence models, in which the scalar field possesses a $O(N)$ internal symmetry. For the $O(N)$ quintessence models discussed above, the angular contribution are not negligible at the early epoch of the universe. While with the time evolving, the potential of the tracker type $O(N)$ quintessence will spill out the angular momentum because the amplitude R becomes very large. However, we have shown that the introduction of "angular" component will not alter the tracker property of the tracker-type quintessence. While for the cosine-type $O(N)$ quintessence, the amplitude of R does not necessarily goes to be very large and therefore the angular contribution can not be neglected even in the current epoch of the universe. We have shown that some tracker-type $O(N)$ quintessence is possibly unstable against Q ball formation while the others are stable. Therefore, it is very important to study the cosine-type $O(N)$ quintessence and the corresponding CMB anisotropy caused by it, which we will discuss in another preparing work.

We also would like to point out that the interaction between $O(N)$ quintessence and other scalar fields could be of certain significance in the formation of cold dark matters. that is, after the monopoles were produced during the phase transition in the early universe, the $O(2)$ quintessence might be absorbed into the monopoles to form a new type of cold stars which are possible candidates for dark matters. We left the detailed discussion in another work in preparation.

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- [1] de Bernardis P et al 2000 Nature **404** 955
Hanany S et al 2000 Astrophys. J. **545** 1
 - [2] Bahcall N, Ostriker J P, Perlmutter S and Steinhardt P J 1999 Science **284** 1481
 - [3] Perlmutter S et al 1999 Astrophys. J. **517** 565
Riess A G et al 1998 Astron. J. **116** 1009 astro-ph/9805201
 - [4] Ratra B and Peebles P J 1988 Phys. Rev. **D37** 3406
 - [5] Caldwell R R, Dave R and Steinhardt P J 1998 Phys. Rev. Lett. **80** 1582
 - [6] Steinhardt P J, Wang L and Zlatev I 1999 Phys. Rev. **D59** 123504 astro-ph/9812313
 - [7] Coble K, Dodelson S, Frieman J 1997 Phys. Rev. **D55** 1851
 - [8] Kim J E 1999 JHEP **9905** 022
Nomura Y, Watari T and Yanagida T 2000 Phys. Lett. **B484** 103
Nomura Y, Watari T and Yanagida T 2000 Phys. Rev. **D61** 105007
 - [9] Chiba T 2001 Phys. Rev. **D64** 103503
 - [10] Masiero A, Pietroni M and Rosati F 2000 Phys. Rev. **D61** 023504
Barreiro T, Copeland E J and Nunes N J 2000 Phys. Rev. **D61** 127301
Copeland E J, Nunes N J and Rosati F 2000 Phys. Rev. **D62** 123503
 - [11] Balbi A et al. 2001 Astrophys. J. **547** L89
 - [12] Corasaniti P S and Copeland E J astro-ph/0107378 and the references therein
 - [13] MAP homepage, <http://map.gsfc.nasa.gov/>
 - [14] PLANCK homepage <http://astro.estec.esa.nl/SA-general/Projects/Planck>
 - [15] Boyle L A, Caldwell R R and Kamionkowski M astro-ph/0105318
 - [16] Gu Je-An and Hwang W-Y P astro-ph/0105099.
 - [17] Copeland E J, Liddle A R and Wands D 1998 Phys. Rev. **D58** 4686
 - [18] Kusenko A, Shaposhnikov M 1998 Phys. Lett. **B418** 46
Kasuya S and Kawasaki M 2000 Phys. Rev. **D61** 041301
Kasuya S and Kawasaki M 2000 Phys.Rev. **D62** 023510
 - [19] Kasuya S astro-ph/0105408
 - [20] Coleman S 1985 Nucl. Phys. **262** 263
 - [21] Zlatev I, Wang L and Steinhardt P J 1999 Phys. Rev. Lett. **82** 896 astro-ph/9807002
 - [22] Skordis C and Albrecht A astro-ph/0012195
 - [23] Bento M C, Bertolami O and Santos N C astro-ph/0106405